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AN UPDATED PERIOD ANALYSIS FOR AC BOOTIS

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AC Bootis (=BD +46°2004 = TYC 3474-905-1 = HIP 73103, Sp. F8V) was discovered to be variable by Geyer (1955, as reported by Mauder, 1964). The system was identified as a member of the W-UMa class by Zessewitch (1956), and the correct period first determined by Mauder (1964). Since then, numerous photoelectric light curves have been obtained, and variations over time noted. Several light curve analyses have been preformed using various codes, most recently those of Wilson-Devinney (Wilson & Devinney, 1971, Wilson, 1990; hereafter WD). (See Nelson, 2010 for a more complete set of references.) Radial velocity curves were obtained by Hrivnak (1993) using a cross-correlation technique. Nelson (2010) performed a full WD analysis (using radial velocities and multi-filter CCD light curves determined by him), solving for the fundamental parameters. Independently, Alton (2010) presented a photometric WD analysis of his own light curve data and also those from five other authors, arriving at a unified model with closely similar values for the parameters, differing only in those for the spots, which varied over time.

Both authors (Nelson and Alton) also undertook separate period analyses, concluding that the period had changed over the interval from 1929 (cycle −54880) to 2012 (cycle 31000). Based on visual, photoelectric and CCD eclipse timings, Nelson (2010) concluded that the period was constant from 1929 to 1982, after which there was a "sudden rise in the period; after that, the period displayed a slow, steady increase over time". He suggested that the abrupt change in period could be explained by an episodic mass interchange possibly as the two stars established contact. After that, there was a continuous period increase at a constant rate. See Fig. 1.

Alton (2010) also performed a period analysis, noting a "continual increase in orbital period over the last 48 years or longer, thereby suggesting an ongoing exchange of mass. Fourier analysis also revealed possible periodicity in $O-C$ residuals which was "heavily influenced by a putative sinusoidal-like wave most apparent over the past twenty years." He gave the opinion that the variation was not due to an unseen component but rather due to spot formation on either stellar component.

The system was also discussed briefly in the review paper by Nelson et al. (2014).

A new period analysis has now been completed allowing for the light time effect (LTE) due to a possible third star. The full set of equations for period change study were given in Nelson (2015a), but two are reproduced here:

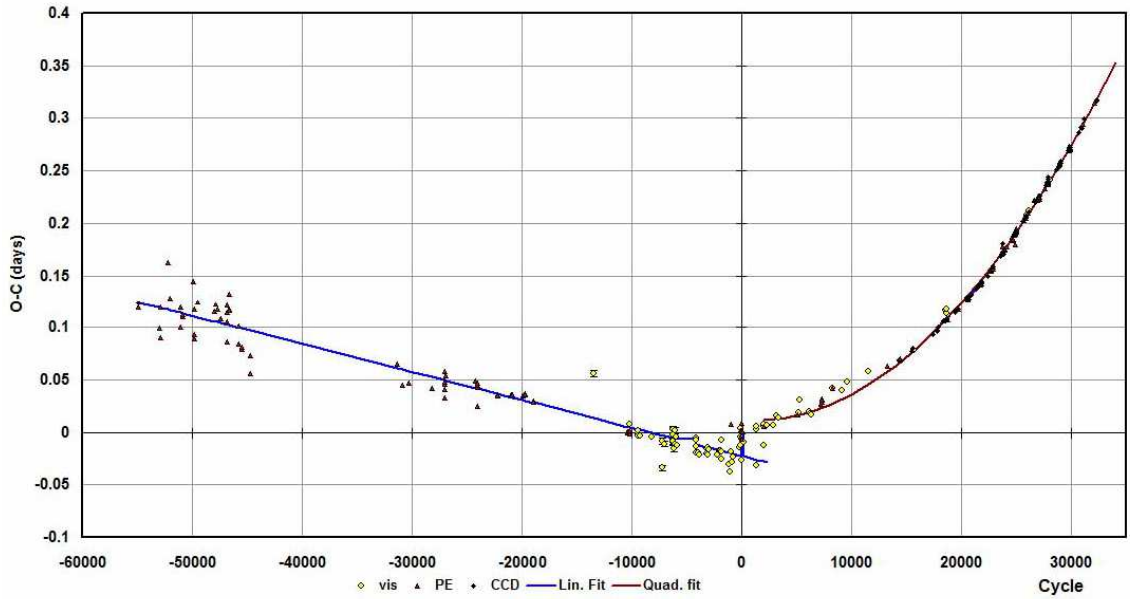

Figure 1. AC Boo – ET diagram (Nelson, 2010)

The equation of a best-fit parabola in the eclipse timing diagram is:

$$Y_1 = c_0 + c_1 n + c_2 n^2 \tag{1}$$

where n is the cycle number. The equation for the difference in time due to the light time effect (Irwin 1952, 1959) is:

$$Y_2 = A\left[\frac{1-e^2}{1+e\cos\nu}\sin(\nu+\omega) + e\sin\omega\right] \tag{2}$$

where A = semi-amplitude in days (= $a_{12}\sin i/c$), a_{12} = semi-major axis of the 1-2 pair about the centre of mass of the triple system, c = speed of light = 300 000 km/s, e = eccentricity of the (1,2)-3 orbit, ν = the true anomaly (varies), ω = argument of periastron (constant). Additional parameters are P_3 = period of the (1,2)-3 system and T_0 = time of periastron passage.

As in Nelson (2010), the elements of Schieven et al. (1983) were used:

$$\text{J.D.Hel.minI} = 24\,45117.781(1) + 0.3524321(2)n \tag{3}$$

A note about the method used to obtain a solution may be of use. In the present study, all of the available eclipse timings were entered into an Excel worksheet, each with the standard weighting $w_i \sim 1/e_i^2$. Analysis of deviations from the curve of best fit (see Fig. 3) yielded weights of 0.02 for group 1 (cycle < -40000), 0.1 for group 2 ($-40000 <$ cycle < -10000) and 1 for PE/CCD data for group 3 (cycle > -15000). (The visual data were initially given weights of 0.1 but were eventually excluded from the fit.)

All eight parameters (c_0, c_1, c_2, A, e, ω, P_3, and T_0) were listed in adjacent cells, and additional equations given in Nelson (2015a) were used to compute expected values $C = Y_1 + Y_2$ for each row in the worksheet, which corresponded to one data point. The weighted sum of the differences squared, $\sum w_i(O-C)^2$ was then minimized using the 'Solver' tool.

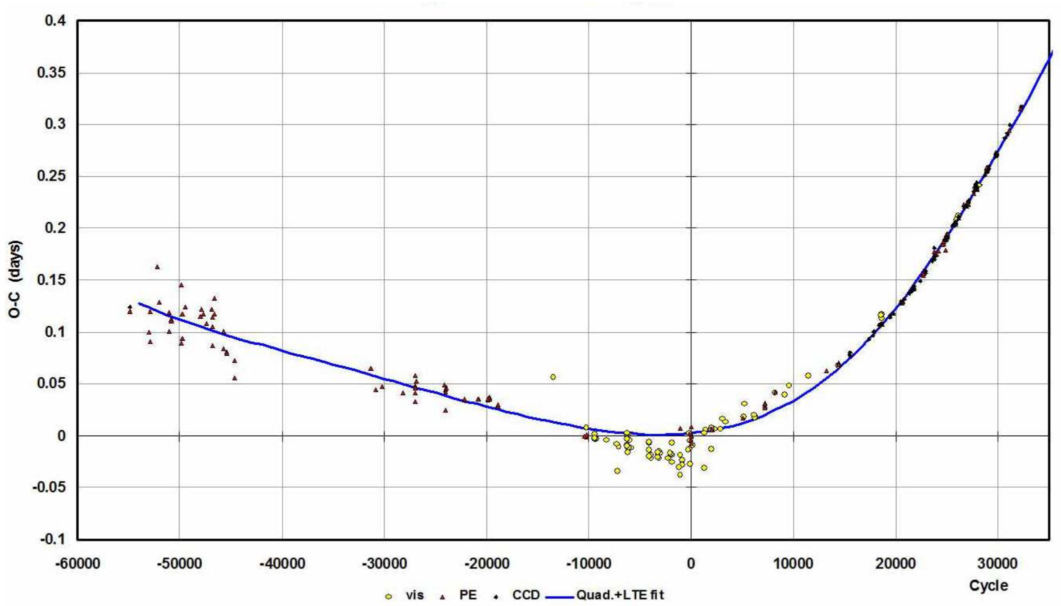

Figure 2. AC Boo – ET diagram

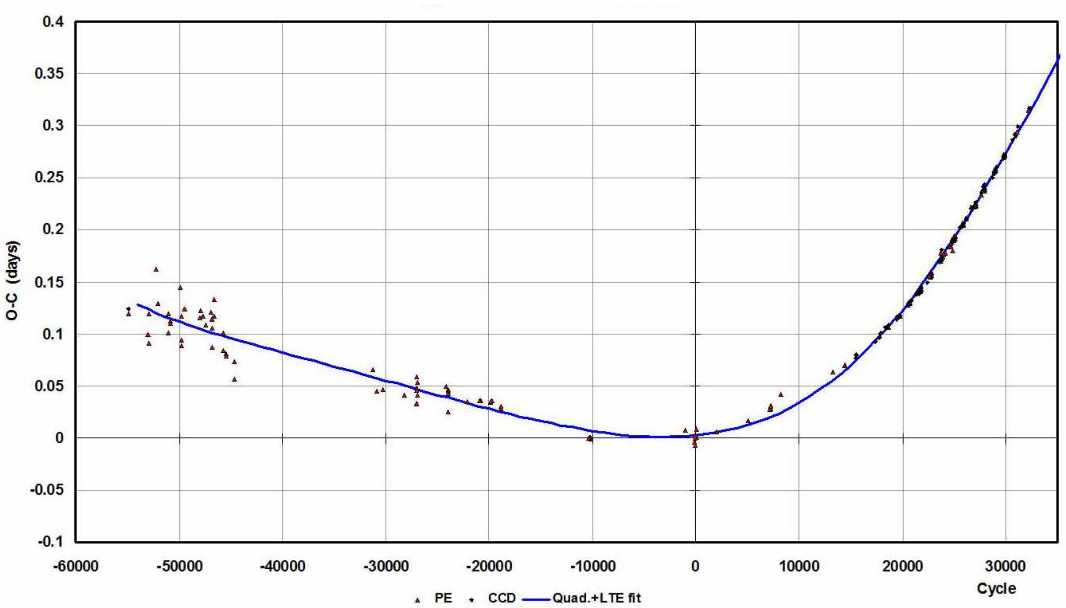

Figure 3. AC Boo – ET diagram

Table 1: Parameters for the quadratic + LTE fit for the eclipse timing differences of AC Bootis

Quantity	Coeff.	error	Unit
Constant, c_0	2.4	0.7	10^{-2} days
Slope, c_1	4.26	0.06	10^{-6} days/cycle
Quadratic coefficient, c_2	1.28	0.11	10^{-10} days^2/cycle
dP/dt	2.67	0.23	10^{-7} days/year
Amplitude, $A = (a_3 \sin i)/c$	0.047	0.004	days
$a_{12} \sin i$	8.11	0.54	AU
Eccentricity, e	0.35	0.05	—
Period, P_3	72.4	2.5	years
Argument of periastron, ω	348	11	degrees
Periastron time, T_p	53710	2154	HJD$-$2400000
Mass function, $f(m_3)$	0.10	0.02	$M_\odot$

It was noted that, to the eye, the first solution did not match all the data very well (see Fig. 2). The problem seemed to lie with the visual data. Using only photoelectric and CCD data led to a much better fit visually (see Fig. 3).

The residuals from the LTE fit are displayed in Fig. 4 along with the fitted quadratic function. The residuals from the quadratic fit are displayed in Fig. 5 along with the fitted LTE function.

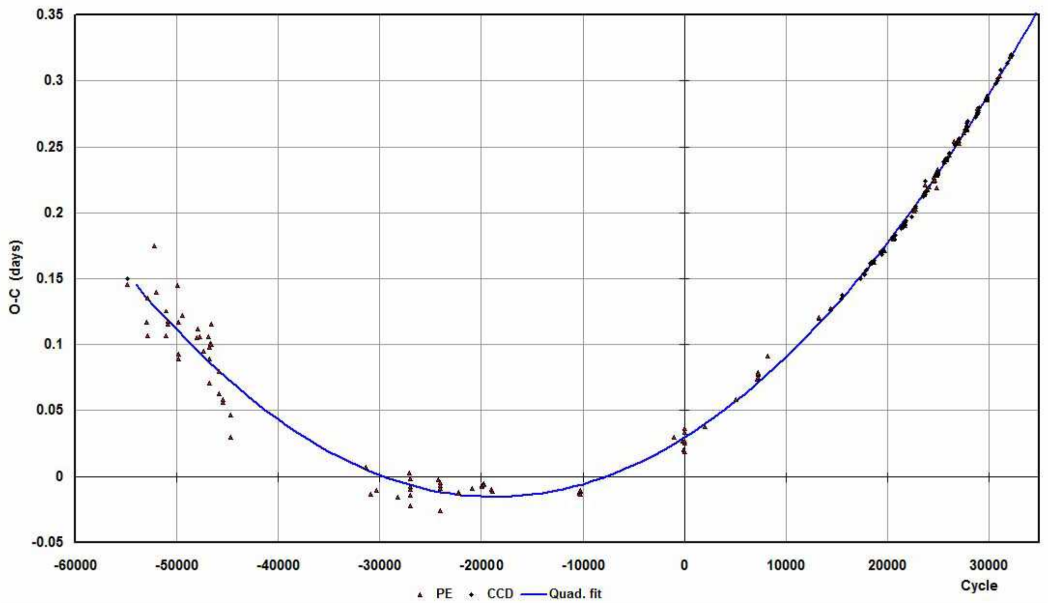

Figure 4. AC Boo – residuals from the LTE fit

The best-fit parameter set is given in Table 1. The procedure used in the error analysis was described in Nelson (2015a).

Nelson (2010) derived masses $m_1 = 0.36(3)\ M_\odot$, $m_2 = 1.20(5)\ M_\odot$ for the system. Using the well-known equation

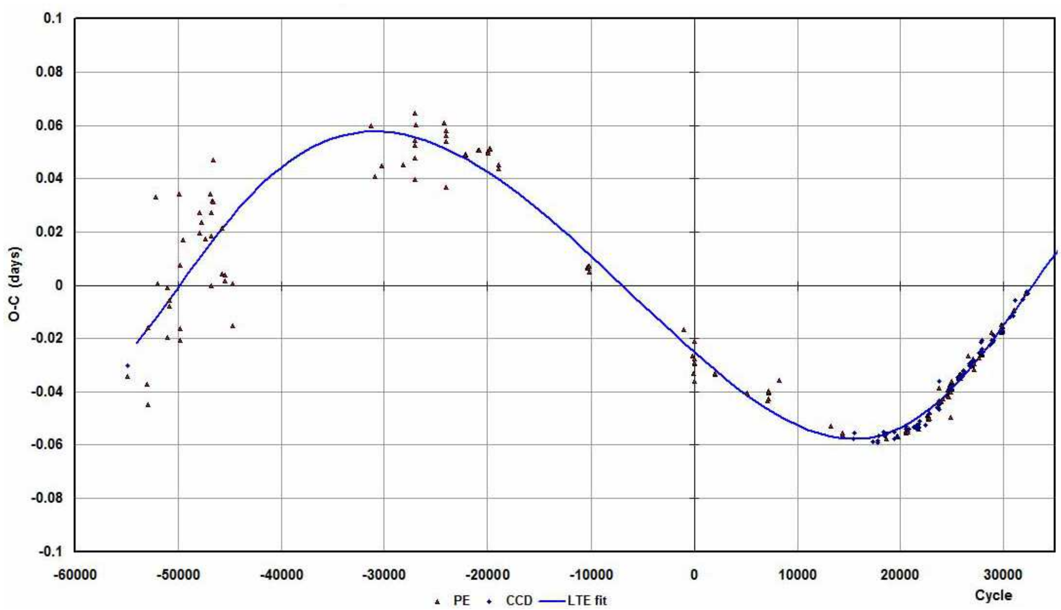

Figure 5. AC Boo – residuals from the quadratic fit

$$\frac{dm_1}{dt} = \frac{1}{3P(\frac{1}{m_2} - \frac{1}{m_1})} \frac{dP}{dt} \tag{4}$$

and the value for dP/dt in Table 1, one obtains the mass transfer rate $dm_1/dt = (-1.3 \pm 0.3) \times 10^{-7}\ M_\odot$/year. Since the system is of the W-type subclass (the star having the lesser mass is the brighter and is therefore eclipsed at the primary eclipse and is designated as star 1), this means that the more massive star is gaining mass at the expense of the less massive one.

Obtaining reliable values for the mass exchange rates in overcontact eclipsing binaries is problematic. This is because both magnetic cycles (Applegate, 1992) and the light time effect can mimic the quadratic function in the eclipse timing difference $(O - C)$ plot occurs when mass is exchanged at a constant rate.

This topic, dealt with in review papers by Nelson et al. (2014, 2015a, 2015b), is all the more important because of the finding by Pribulla & Rucinski (2006) that most contact binary stars exist in multiple systems. Therefore, one might expect the light time effect to be common with such systems.

However, if it is true that the light time effect as modelled above adequately explains the somewhat complex (viz. non-linear or quadratic) features in the ET plots of Figs. 1-3 and that magnetic cycles can be ruled out, then the residuals, as displayed in Fig. 4, are due entirely to mass exchange, and the value $dm_1/dt = (-1.30 \pm 0.27) \times 10^{-7}\ M_\odot$/year is reliable.

It is noted, as stressed in Nelson et al. (2014, 2015a, 2015b), that subsequent eclipse timing data will often demand a new fit. Sometimes the fit is completely wrong. In a sense all fits are tentative, to some level of uncertainty. In any case, the results in Table 1 should be treated with caution, especially since little more than one period of the putative third star has been observed. The error estimates are mathematically sound, but real errors may be larger. On the other hand, repeated tests with existing data have failed to reveal any other plausible values for P_3 (and hence the other parameters).

The $O - C$ file for this system may be obtained at the URL given below in Nelson (2015b).

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