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COMMENTS ON THE P-L-C RELATION OF THE CLASSICAL CEPHEIDS

The problem of P-L-C relation for classical Cepheids:

$$M_{<V>} = a \log P + b<B-V>_o + c \qquad (1)$$

is discussed nowadays for the reason of various numerical values proposed for the colour term coefficient b and because of many critical remarks concerning the methods of calculation of this quantity, see Clube and Dave (1983). Here we present the arguments in favour of a large value of the colour term coefficient for galactic and LMC Cepheids in accordance with Brodie and Madore's (1980) results.

1. The paper by Fernie (1984) contains the list of Cepheid radii obtained by Wesselink method. The $<B-V>_o$ for these stars have been obtained from the $<B-V>$ values taken from a catalogue by Schaltenbrand and Tammann (1971) and reddenings from Dean et al. (1978) and Pel (1978). On the basis of these data we got for 20 most reliable radii and log P<1 the following P-R-C relation:

$$\log R = \underset{\pm 0.113}{1.029} \log P - \underset{\pm 0.181}{0.572} <B-V>_o + \underset{\pm 0.077}{1.259} \qquad (2)$$

$$\text{s.d.} = 0.042$$

The small value of the colour term coefficient -0.572 justified the application in this case of the standard least squares method, which due to the narrow range of $<B-V>_o$ and its correlation with log P leads to a systematic lowering of greater values of coefficients. In order to pass from log R to the absolute magnitudes, $M_{<V>}$, we use the formula:

$$M_{<V>} = -5 \log R + S_V, \qquad (3)$$

where S_V is the surface brightness:

$$S_V = 42.312 - 10 \log T_e - \text{B.C.} \qquad (4)$$

According to van Genderen (1983), for Cepheids

$$B.C. = 0.430 - 0.603 \langle B-V\rangle_o \quad (5)$$

and $$\log T_e = 3.870 - 0.175 \langle B-V\rangle_o \quad (6)$$

So we got

$$S_V = 3.182 + 2.353 \langle B-V\rangle_o \quad (7)$$

and the P-L-C relation:

$$M_{\langle V\rangle} = -5.295 \log P + 5.213 \langle B-V\rangle_o - 3.115 \quad (8)$$

From this example we see that the small values for b are not acceptable because of significant dependence of S_V on $\langle B-V\rangle_o$. Therefore b should be greater than 2.353 and in this case amounts to 5.213.

2. As the next group of stars we consider the long period Cepheids in LMC. The following numerical data have been taken from the paper by van Genderen (1983): log P, V_{Jo} which we assume to be equal to $M_{\langle V\rangle}$ + Mod, and $(B-V)_{Je}$ instead of $\langle B-V\rangle_o$. In this case we use the following procedure, avoiding the least squares method:

Eq. (1) means that the Cepheids are placed on the plane in the three dimensional space: log P-$M_{\langle V\rangle}$ - $\langle B-V\rangle_o$. We divide the group of the investigated stars into two halves with shorter and longer periods and calculate for both groups the mean values: log P_1, $M_{\langle V\rangle 1}$, $\langle B-V\rangle_{o1}$ and log P_2, $M_{\langle V\rangle 2}$, $\langle B-V\rangle_{o2}$. We assume that the points with the coordinates so obtained are placed also on the plane defined by eq. (1). The projections of these points on the log P-$M_{\langle V\rangle}$ plane have the coordinates log P_1, $M_{\langle V\rangle 1}$ and log P_2, $M_{\langle V\rangle 2}$ and they determine the P-L relation as the straight line:

$$M_{\langle V\rangle} = g \log P + h \quad (9)$$

The individual deviations of stars from this line:

$$\Delta M_{\langle V\rangle} = M_{\langle V\rangle} - g \log P - h \quad (10)$$

are due to the arrangement of stars on the P-L-C plane and should not be treated only as errors. On the contrary, their existence is a proof of reality of the plane defined by eq. (1).

Similarly we got the P-C relation as a straight line

$$\langle B-V\rangle_o = d \log P + e \quad (11)$$

on the log P-$\langle B-V\rangle_o$ plane passing through the points log P_1, $\langle B-V\rangle_{01}$ and log P_2, $\langle B-V\rangle_{02}$. So it is possible to calculate the similar deviations for individual stars:

$$\Delta\langle B-V\rangle_o = \langle B-V\rangle_o - d \log P - e \tag{12}$$

From the geometry of this problem it follows that the deviations $\Delta M_{\langle V\rangle}$ and $\Delta\langle B-V\rangle_o$ and the quantities occurring in eqs. (1), (9), and (11) are related as follows:

$$\Delta M_{\langle V\rangle} = b\Delta\langle B-V\rangle_o, \tag{13}$$

$$a = g - b\,d, \qquad c = h - be \tag{14}$$

This method applied to 19 long-period Cepheids in LMC according to van Genderen's paper (1983) led to the following results:

$$V_{Jo} = M_{\langle V\rangle} + \mathrm{Mod} = -2.516 \log P + 16.413,$$

$$(B-V)_{Je} = \langle B-V\rangle_o = 0.472 \log P + 0.078$$

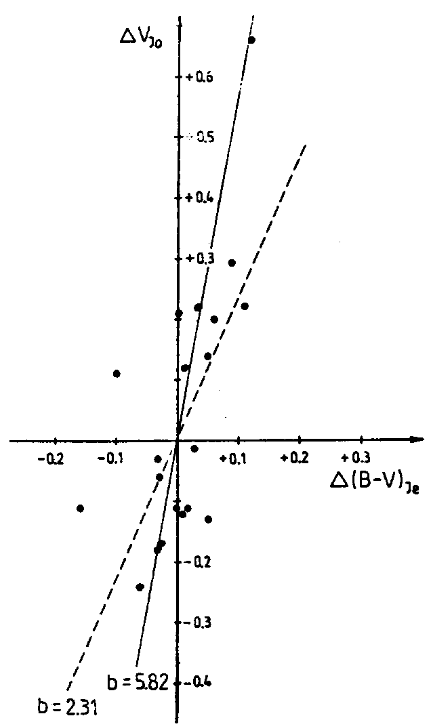

Figure 1

Determination of the coefficient b for the LMC Cepheids

The deviations $\Delta V_{Jo} = \Delta M_{<V>}$ and $\Delta(B-V)_{Je} = \Delta<B-V>_o$ are plotted in Figure 1. Using again the mean values of these quantities for positive and negative $\Delta M_{<V>}$ we got from eq. (13)

$$b = 5.82$$

and from eqs. (14): a = −5.26 and c + Mod = 15.96. The same numerical data, treated by standard least squares method, led to the values:

$$a = -3.474, \quad b = 2.307, \quad c + Mod = 16.040.$$

As it was stated above in this case the least squares method gives significantly lower value for the coefficient b. But, as is shown in Figure 1, b = 2.307 does not suit with the observational points.

3. Finally the $M_{<V>}$ and $<B-V>_o$ values for 51 galactic Cepheids with $<B-V>_o < 0.85$ have been taken from the author's paper, (Opolski, 1982) and subjected to the same method as LMC stars. The results are as follows:

$$M_{<V>} = -2.635 \log P - 1.971,$$

$$<B-V>_o = 0.424 \log P + 0.304,$$

$$a = -5.310, \qquad b = 6.31, \qquad c = -3.457$$

In order to get in this problem the results without systematic errors introduced by the least squares method it is enough to change the form of the P-L-C relation to:

$$<B-V>_o = x \log P + y\, M_{<V>} + z \tag{15}$$

In this case the small numerical values of the coefficients x and y and the greater range of the $M_{<V>}$ variability allow to get proper results using the least squares method. In this way for stars in the LMC taken again from van Genderen's paper (1983) we got:

$$(B-V)_{Je} = \underset{\pm 0.130}{0.884} \log P + \underset{\pm 0.051}{0.173}\, V_{Jo} - \underset{\pm 0.839}{2.726}$$

$$s.d. = 0.048$$

From this we have:

$$V_{Jo} = -5.106 \log P + 5.774(B-V)_{Je} + 15.739,$$

whereas the application of the same method directly to the eq. (1) gives:

$$V_{Jo} = \underset{\pm 0.357}{-3.474} \log P + \underset{\pm 0.686}{2.307}(B-V)_{Je} + \underset{\pm 0.257}{16.040}$$

$$s.d. = 0.176$$

But this solution has a systematic error. The differences between the observed and calculated V_{Jo} are correlated with these quantities. The negative differences are predominating in the range of smaller V_{Jo} while the positive ones are connected mostly with larger V_{Jo}. Therefore they do not have the character of accidental errors.

Similarly for galactic Cepheids (Opolski, 1982) we got:

$$\langle B-V\rangle_o = \underset{\pm 0.095}{0.982} \log P + \underset{\pm 0.035}{0.191} M_{\langle V\rangle} + \underset{\pm 0.076}{0.646}$$

$$\text{s.d.} = 0.054$$

or $$M_{\langle V\rangle} = -5.141 \log P + 5.238\langle B-V\rangle_o - 3.384$$

For this case eq. (1) gives directly:

$$M_{\langle V\rangle} = \underset{\pm 0.206}{-3.548} \log P + \underset{\pm 0.367}{1.998}\langle B-V\rangle_o - \underset{\pm 0.131}{2.540},$$

$$\text{s.d.} = 0.176,$$

with the similar systematic error as in the foregoing example. Also the result obtained by Martin et al. (1979) for 26 stars in the LMC achieved by the maximum likelihood fit: $a = -3.80$, $b = 2.70$, $c + \text{Mod} = 16.41$, is encumbered with the systematic errors.

It is worth to notice that nine SMC Cepheids, (van Genderen, 1983), give more consistent results:

$$(B-V)_{Je} = \underset{\pm 0.151}{1.349} \log P + \underset{\pm 0.063}{0.338} V_{Jo} - \underset{\pm 1.032}{5.789}$$

$$\text{s.d.} = 0.038$$

or $$V_{Jo} = -3.988 \log P + 2.957(B-V)_{Je} + 17.119$$

The relation obtained directly from eq. (1) for these stars is the following:

$$V_{Jo} = \underset{\pm 0.283}{-3.702} \log P + \underset{\pm 0.454}{2.454}(B-V)_{Je} + \underset{\pm 0.220}{17.009}$$

$$\text{s.d.} = 0.101$$

In this case the smaller value of the coefficient b causes that the lowering from 2.957 to 2.454 is not so significant as in the other examples.

A. OPOLSKI and T. CIURLA
Wroclaw University Observatory
51-622 Wroclaw, Poland

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