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DETERMINATION OF CEPHEID DISTANCES BY MEANS OF WESENHEIT FUNCTION

It is well known that the determination of Cepheid distances is complicated by the necessity of taking into account the effect of the interstellar absorption. Here we propose a method for determination of true distances when only the ratio R of total A_V to selective absorption E_{B-V} is known:

$$R = A_V/E_{B-V}$$

To solve this problem we use the Wesenheit function W introduced by van den Bergh (1968) and discussed by Madore in a series of papers, e.g. Madore (1982). The important property of this function is its independence on interstellar absorption:

$$W = \langle V\rangle_o - R\langle B-V\rangle_o = \langle V\rangle - R\langle B-V\rangle \qquad (1)$$

where $\langle V\rangle = \langle V\rangle_o + A_V$ and $\langle B-V\rangle = \langle B-V\rangle_o + E_{B-V}$.

A similar function for absolute magnitude $M_{\langle V\rangle}$ has been defined by Madore (1976):

$$W_M = M_{\langle V\rangle} - R\langle B-V\rangle_o \qquad (2)$$

This expression can be compared with the P-L-C relation for classical Cepheids pulsating in fundamental mode:

$$M_{\langle V\rangle} = a\cdot \log P_F + b\langle B-V\rangle_o + c \qquad (3)$$

From Eqs. (2) and (3) we have:

$$W_M = a\cdot \log P_F + (b-R)\langle B-V\rangle_o + c \qquad (4)$$

It was stated many times that the numerical value of the coefficient b is very poorly determined by the observational data. Even the method of calculation, least squares or maximum likelihood, may influence significantly the value of this coefficient, as is shown in the paper by Martin, Warren and Feast (1979) where quantities from 2.18 to 3.08 have been obtained. The reason is that b determines the slope of constant P lines on the $M_{\langle V\rangle} - \langle B-V\rangle_o$ plane. These lines are nearly perpendicular to the axis of rather narrow stripe occupied by classical Cepheids and it is difficult to fix the accurate value of the slope b.

Taking into account this fact we propose to assume that b equals R, for which we accept the value 3.20 according to Seaton (1979):

$$b = R = 3.20 \quad .$$

This makes W_M depending only on P_F :

$$W_M = a \cdot \log P_F + c \tag{5}$$

This way we take advantage of the accidental, numerical similarity of two dimensionless quantities: R describing the optical property of the interstellar matter and b appearing in the P-L-C relation, when V and B photometric systems are used.

To illustrate the result of the proposed procedure we have calculated P-L-C and P-W_M relations by means of the data of the author's paper (Opolski, 1982, Table II) where the values $\log P_F$, $M_{\langle V\rangle}$ and $\langle B-V\rangle_o$ for 66 Cepheids are listed. The results achieved by the least squares method are as follows:

$$M_{\langle V\rangle} = -3.8601 \log P_F + 2.731\langle B-V\rangle_o - 2.782 \tag{6}$$

s.d.= ± 0.202

$$W_M = M_{\langle V\rangle} - 3.20\langle B-V\rangle_o = -4.0988 \log P_F - 2.897 \tag{7}$$

s.d.= ± 0.205 .

Comparing the standard deviations in these formulae we see that the accuracy of the three parameter relation for $M_{\langle V\rangle}$ with b = 2.731 is practically the same as that for W_M with two parameters where we constrain the coefficient b to be equal to 3.20 .

This fact makes it easy to calculate the true distance moduli avoiding the direct taking into account the interstellar absorption. From Eqs. (1) and (2) we have:

$$W - W_M = \langle V\rangle_o - M_{\langle V\rangle} = Mod_W = 5 \cdot \log r_W - 5 \tag{8}$$

where Mod_W is the distance modulus and r_W the true distance in parsecs. Now taking advantage of the properties of the functions W and W_M , Eqs. (1) and (5), we get the relations:

$$Mod_W = W - W_M = \langle V\rangle - 3.20\langle B-V\rangle - a \log P_F - c \tag{9}$$

$$\log r_W = 0.2[\langle V\rangle - 3.20\langle B-V\rangle - a \log P_F - c] + 1 \tag{10}$$

where only directly observed quantities P_F , V and ⟨B-V⟩ are used.

The numerical values for a and c in Eqs. (9) and (10) can be taken from the formula (7). But we can also calculate W_M for calibration Cepheids, members of clusters or associations for which Mod are known:

$$W_M = W - Mod = \langle V\rangle - 3.20\langle B-V\rangle - Mod \tag{11}$$

and then we can fix the $P_F - W_M$ relation. From the published moduli of cali-

Table I. Distance moduli Mod and function W_M for calibration Cepheids

	log P	<V>	<B-V>	Mod	W_M
SU Cas	0.290	$5^m.969$	$0^m.725$	7.76	-4.11
EV Sct	0.490	10.128	1.157	11.29	-4.86
CE Cas b	0.651	10.988	1.140	12.79	-5.45
CF Cas	0.687	11.110	1.227	12.79	-5.61
CE Cas a	0.711	10.919	1.213	12.79	-5.75
UY Per	0.730	11.306	1.591	12.16	-5.94
CV Mon	0.731	10.296	1.365	11.37	-5.44
U Sgr	0.828	6.714	1.142	9.24	-6.18
DL Cas	0.903	8.942	1.216	11.54	-6.49
S Nor	0.989	6.414	0.969	10.02	-6.71
VX Per	1.037	9.300	1.242	12.16	-6.83
SZ Cas	1.134	9.826	1.505	12.16	-7.15
VY Car	1.277	7.446	1.193	11.73	-8.10
RU Sct	1.294	9.500	1.779	11.76	-7.95
RZ Vel	1.310	7.114	1.209	11.39	-8.14
SW Vel	1.371	8.126	1.252	12.25	-8.13
RS Pup	1.617	7.006	1.489	11.56	-9.32
SV Vul	1.654	7.221	1.534	11.77	-9.46
S Vul	1.826	9.00	1.92	13.38	-10.52

bration Cepheids we have selected 19 stars listed in Table I. The moduli have been corrected to one system based on the Hyades modulus Mod = 3.31 according to Hanson (1978). The $P_F - W_M$ relation achieved by the least squares method is as follows:

$$W_M = -4.0455 \log P_F - 2.797 \qquad (12)$$

s.d. = ±0.158.

This result is very similar to Eq. (7) but has better accuracy and we propose it as final. So for galactic classical Cepheids pulsating in the fundamental mode we have distance moduli Mod_W and distances r_W expressed by the formulae:

$$Mod_W = \langle V\rangle - 3.20\langle B-V\rangle + 4.0455 \log P_F + 2.797 \qquad (13)$$

$$\log r_W = 0.2\langle V\rangle - 0.64\langle B-V\rangle + 0.8091 \log P_F + 1.559 \qquad (14)$$

With the aid of these relations it is possible to calculate Mod_W and r_W for a large number of Cepheids, e.g. contained in the catalogue by Schaltenbrand and Tammann (1971). Such an investigation is in progress.

An attempt to apply Eq. (14) to galactic Cepheids was made by comparing log r calculated by means of the standard relation:

$$\log r = 0.2[\langle V\rangle - M_{\langle V\rangle} + 5 - 3.20\, E_{B-V}] \qquad (15)$$

with $\log r_W$ resulting from Eq. (14). The data needed for these calculations have been taken from papers: <V> and <B-V> — Schaltenbrand and Tammann (1971), $M_{\langle V\rangle}$ — Opolski (1982), E_{B-V} — Dean, Warren and Cousins (1978). As the result we got the mean difference $\Delta \log r = \log r - \log r_W = +0.0286 \pm 0.0044$ and standard deviation of an individual star ±0.038 .

Application of Eq. (13) to the extragalactic Cepheids in the Magellanic Clouds with the photometric data taken from Gascoigne (1969) and Madore (1975) for stars with periods $0.3 < \log P_F < 2.2$ led to the following results:

LMC - 33 stars: $Mod_W = 19.07 \pm 0.21$, $r_W = 65.2 \pm 6.6$ kpc

SMC - 32 stars: $Mod_W = 19.79 \pm 0.18$, $r_W = 90.8 \pm 7.8$ kpc

The details of these investigations will be published in a separate paper.

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