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ANALYSIS OF GRADIENT DIAGRAM FOR CEPHEIDS

As was found by E.S.Kheylo [1] all cepheids occupy a band on the plane (Δ_U, ΔV), where $\Delta_U = dU/dB$, $\Delta_V = dV/dB$, the band being elongated along the axis Δ_U. We shall call this plane the gradient diagram. Inside the band all cepheids separate according to their types. This paper is an attempt to explain such a location of cepheids on the gradient diagram.

Let us consider the light variations of cepheids to be connected with changes of dimension and of the black body temperature. In this case we can write for the luminosity in the Q-region

$$I_Q \sim R^2 I_Q(T), \qquad Q = U,B,V \qquad (1)$$

where R denotes the radius of star, I_Q (T) the radiation intensity in the isophotal wave length of Q for temperature T. Isophotal wave lengths were adopted to be 3680A for U, 4450A for B and 5460A for V [2]. The Wien formula gives satisfactory accuracy for $I_Q(T)$.

For radius changes from R_1 to R_2 and temperature changes from T_1 to T_2, equation (1) gives for the gradients

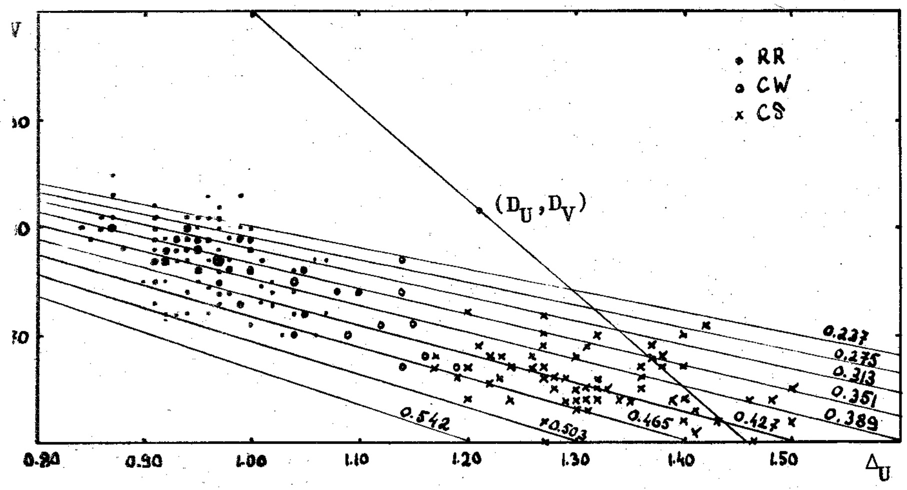
$$\Delta_U = \frac{a-C_U\ d}{a-C_B\ d}, \qquad \Delta_V = \frac{a-C_V\ d}{a-C_B\ d}, \qquad (2)$$

where $a = 2 \ln R_1/R_2 \quad \Delta R/R_2$ when changes of radius are small, $d = (T_2-T_1)/T_1T_2 = \Delta T/T_1T_2 \cdot c_U=3.91 \cdot 10^4$, $c_B=3.23 \cdot 10^4$, $c_V = 2.64 \cdot 10^4$. Educations (2) indicate, that Δ_U and Δ_V are linearly connected. This line passes through the point (1,1) and the point

$$D_U = 1.21, \qquad D_V = 0,815, \qquad (3)$$

which corresponds to the changes of the black body temperature only. In the Figure one can see that the cepheids are located below this line. Thus it is impossible to explane light changes of cepheids by the changes of dimension and of the black body temperature.

The changes of R or T in the U-region may be assumed to differ from those in the B- and V-regions. In this case $d_U \neq d_{B,V}$ or $a_U \neq a_{B,V}$.



Lines of equal m and the band of cepheids

Two hypotheses were considered

$$d_U \neq d_B = d_V \,, \qquad a_U = a_B = a_V \tag{4}$$

and

$$d_U = d_B = d_V = d, \qquad a_U \neq a_B = a_V \tag{5}$$

Better agreement seems to be achieved for the location of cepheids in the gradient diagram in the case (5). Then

$$\Delta_U = D_U \frac{m-1}{D_V-1} (1 - \Delta_V) \,, \tag{6}$$

where $m = a_U/C_U d$ is a free parameter. As follows from the Figure these lines occupy the cepheid band for m=0.23-0.58. Then Δ_U and Δ_V for a cepheid permit to obtain

$$\frac{a_U}{d} = C_U \quad 1 - \frac{\Delta_U}{D_U} \cdot \frac{1 - D_V}{1 - \Delta_V} \quad = \quad C_U \; m \tag{7}$$

and

$$\frac{a_U}{a} = D_U \frac{1 - \Delta_V}{D_V - \Delta_V} - \Delta_U \frac{1 - D_V}{D_V - \Delta_V} \tag{8}$$

Table 1

Star	Type	Δ_U	Δ_V	m	$\frac{a_U}{d}\cdot 10^{-4}$	$\frac{a_U}{a}$
δ Cep	Cδ	1.29	0.64	0.451	1.77	1.13
η Aql	Cδ	1.36	.65	.343	1.14	1.02
RS Cas	Cδ	1.24	.64	.465	1.82	1.18
XY Cas	Cδ	1,27	.75	.223	0.875	1.03
IX Cas	Cδ	1.27	.60	.513	2.01	1.16
ST Tau	CW	1.19	.67	.448	1.76	1.24
AP Her	CW	1.15	.71	.394	1.54	1.31
CC Lyr	CW	1.04	.75	.363	1.42	1.69
RR Lyr	RR	0.91	.73	.485	1.90	1.30
T Sex	RR	0.87	.80	.335	1.32	5.37
EH Lib	RR	0.97	.77	.355	1.39	2.20

Table 2.

Type	$\bar{\Delta}_U$	$\bar{\Delta}_V$	$\bar{m}$	$\frac{a_U}{d}\cdot 10^{-4}$	$\frac{a_U}{a}$
RR	0.96	0.77	0.362	1.42	2.24
CW	1.13	.71	.405	1.59	1.35
C	1.31	.66	.428	1.68	1.13

Tables 1 and 2 give the result of calculations according to expressions (7) and (8) for cepheids of different types. We came to the following conclusions:

1. The location of cepheids on the gradient diagram can be explained assuming that amplitude of radius changes in the U-region differs from those in the B- and V-regions.

2. It is possible to calculate the ratio of these amplitudes using the results of photoelectric observations eq.(8) . This ratio is the greatest for RR Lyr type and the smallest for classical cepheids.

3. Photoelectric observations permit to obtain the relation between changes of radius and temperature eq.(7) .

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[2]. Landolt-Boernstein. Numerical Data and Functional Relationships in Science and Technology. Group VI. Vol.1. Springer-Verlag. 1965. p.346.