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ANALYSIS OF CEPHEID LIGHT CURVES

The observed light variation of a cepheid variable star in,e.g., V photometric system can be regarded as a sum of two components:

$$\Delta V = \Delta m_v + \Delta m_R \qquad (1)$$

where $\Delta m_v = -2,5 \text{ Log } \frac{F_{v1}}{F_{v2}}$ is due to the variation of radiative flux F_v, and $\Delta m_R = -5 \text{ Log } \frac{R_1}{R_2}$ represents the photometric effect of changes of the radius R. We present here a method to separate these two components.

Let us assume that for each star during the whole period the changes of fluxes: F_V in the V system and F_B in the B system, expressed in terms of Δm_v and Δm_B are in proportion to each other:

$$\Delta m_v = a \, \Delta m_B \qquad (2)$$

From this we have also the proportionality of Δm_v to $\Delta(B - V)$

$$\Delta m_v = k \, \Delta (m_B - m_v) = k \, \Delta (B-V) \qquad (3)$$

with $k = \frac{a}{1-a}$.

This relation can be considered as a generalization of Wesselink's method, which is usually used in the form: for two phases with $\Delta (B-V) = 0$, there is also $\Delta m_v = 0$ and $\Delta V = \Delta m_R$.

According to the formulae (1) and (3) we assume for every two phases the relation:

$$\Delta V = k \quad \Delta (B-V) + \Delta m_R \qquad (4)$$

The photometric observations yield the values (B-V) measured simultaneously with V magnitudes. So the calculation of Δm_V and Δm_R requires only a value of k, which can be obtained from the following considerations.

In most cases the variations of V magnitudes are mainly resulting from the variations of Δm_V, with component Δm_R being less important. So we try to determine the coefficient k, or its mean value over the period, from the condition that for a sufficiently large number of $\Delta m_R = \Delta V - k \; \Delta(B-V)$ we expect to have

$$\sum (\Delta m_R)^2 = \text{min.} \qquad (5)$$

This condition if fulfilled when

$$k = \frac{\sum [\Delta V. \;\; \Delta (B-V)]}{\sum [\Delta (B-V)]^2} \qquad (6)$$

We have applied this method to 29 cepheid variables: 19 Cδ and 10 CW stars. The phases were calculated always from minimum of (B-V), which especially for CW stars may differ from the phase of minimum V (maximum brightness). As an initial phase, the phase of mean value of (B-V) on the descending branch was chosen. In most cases it turned out to be about 0,25. The values of V_o and $(B-V)_o$ for this phase are used to calculate the differences occurring in the formula (6)

$$\Delta V = V - V_o \; , \quad \Delta (B-V) = (B-V) - (B-V)_o .$$

As a rule 10 to 20 values for uniformly spaced phases, read from the V and (B-V) curves, were sufficient to get the coefficient k. Than the values of $\Delta m_V = k \;\; \Delta (B-V)$ and $\Delta m_R = \Delta V - \Delta m_V$ were calculated.

The Δm_R curves have the shapes similar to the ΔR curves obtained by integrating the radial velocities: flat maximum near 0,3 and sharper minimum near 0,8 phase. The amplitude of Δm_R denoted by A_{mR} is only partly dependent on k value. From the ΔR curves one can get the amplitudes of $\Delta R = R_{max} - R_{min}$. By combining them with A_{mR} one can calculate the mean value of radius R:

$$A_{mR} = 5 \text{ Log } \frac{R_{max}}{R_{min}} = 5 \log \frac{R+0,5 \; \Delta R}{R-0,5 \; \Delta R} \qquad (7)$$

The numerical results are presented in the Table. From these values we can draw the following conclusions:

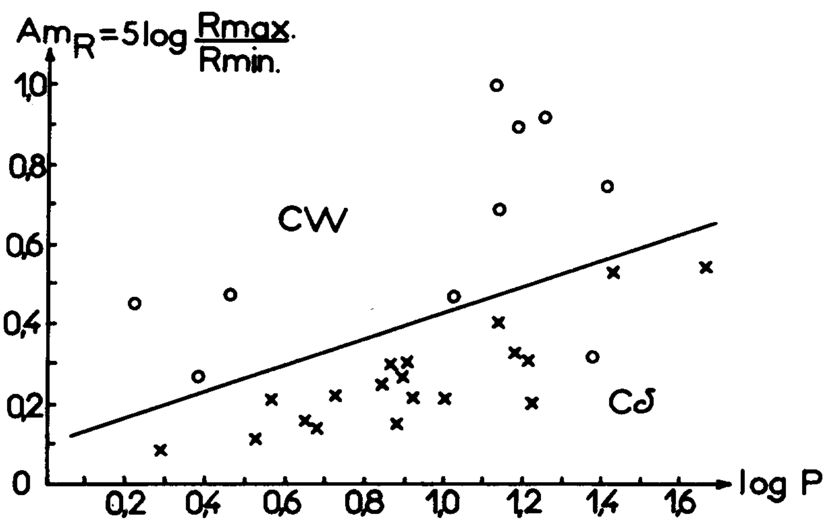
T a b l e

No	Star		log P	Type	k	Am_R	ΔR km	R km
1	SU	Cas	0,290	Cδ	2,13	$0^m,08$	$7,3.10^5$	20.10^6
2	RT	Aur	0,571	Cδ	1,74	0,21	20,8	22
3	T	Vul	0,647	Cδ	1,78	0,16	27,0	37
4	FF	Aql	0,673	Cδ	1,93	0,15	14,7	21
5	δ	Cep	0,730	Cδ	1,62	0,22	40	40
6	U	Aql	0,846	Cδ	1,71	0,25	48	42
7	η	Aql	0,856	Cδ	1,76	0,30	46	33
8	W	Sgr	0,880	Cδ	1,55	0,15	48	70
9	W	Gem	0,898	Cδ	1,38	0,30	48	35
10	S	Sge	0,923	Cδ	1,53	0,22	52	51
11	ζ	Gem	1,006	Cδ	1,21	0,22	48	48
12	TT	Aql	1,138	Cδ	1,37	0,40	116	63
13	X	Cyg	1,216	Cδ	1,48	0,31	173	121
14	Y	Oph	1,233	Cδ	1,64	0,20	51	55
15	T	Mon	1,431	Cδ	1,29	0,53	263	108
16	SV	Vul	1,655	Cδ	0,77	0,54	430	174
17	W	Vir	1,236	CW	2,14	0,92	194	46
18	VZ	Aql	0,222	CW	1,64	0,45		
19	AU	Peg	0,380	CW	1,14	0,27		
20	V465	Oph	0,454	CW	2,41	0,47		
21	V532	Cyg	0,516	Cδ	1,97	0,11		
22	VY	Cyg	0,895	Cδ	1,86	0,27		
23	AP	Her	1,018	CW	1,46	0,47		
24	V1077	Sgr	1,128	CW	0,98	1,00		
25	V410	Sgr	1,139	CW	1,22	0,69		
26	SZ	Cyg	1,179	Cδ	1,35	0,32		
27	AL	Sct	1,192	CW	1,38	0,90		
28	CC	Lyr	1,380	CW	1,53	0,31		
29	TW	Cap	1,407	CW	2,29	0,75		

Photometric data: No 1-16 Mitchell et al.Bol.Obs. Tonantzintla Tacubaya 3, 153, 1964.

No 17-29 Kwee K.Braun L. B.A.N. Sup. Ser. 2,3, 1967.

Radial velocities from different sources.



1. For given P the amplitudes A_{mR} for Cδ stars are systematically smaller than those for CW stars. On the diagramm we can fix the position of line

$$A_{mR} = \frac{1}{3} \text{Log } P + 0,10$$

separating the region of A_{mR} for Cδ from such a region for CW stars. This fact may be used as a purely <u>photometric criterion</u> for classification of Cδ and CW stars. E.g. CC Lyr classified as a CW star, according to our criterion belongs to the Cδ stars.

2. The values of R obtained by means of the foregoing procedure are consistent with the values obtained by other authors using Wesselink's method.

The results for a greater number of stars and a more detailed discussion will be published in the Acta Astronomica.

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