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A REMARK ON THE MULTIPERIODICITY OF SOME PULSATING STARS

In connection with the problem of multiperiodicity of some pulsating stars we propose the interpretation of the observed phenomena by means of modulated oscillations (similar to the modulations introduced in radiotechnique). Let us consider the following simple example:

The oscillation f(t) with the frequency α in the form

$$f(t) = A \cos 2\pi\alpha t$$

is changed by means of modulation with the frequency β to:

$$F(t) = (A+A_1 \cos 2\pi\beta t) \cos 2\pi\left[\alpha t + B\sin 2\pi\beta t\right].$$

This modification is equivalent to the amplitude modulation from A to $A+A_1 \cos 2\pi\beta t$ with the frequency modulation from αt to $\alpha t + B\sin 2\pi\beta t$. When looking for periodicity of such modulated oscillation we must represent it by means of the series:

$$F(t) = a_0 \cos 2\pi\alpha t + a_1 \cos 2\pi(\alpha+\beta)t + a_2 \cos 2\pi(\alpha+2\beta)t + \ldots$$
$$b_1 \cos 2\pi(\alpha-\beta)t + b_2 \cos 2\pi(\alpha-2\beta)t + \ldots$$

where

$$a_0 = J_0(2\pi B)A$$
$$a_1 = J_1(2\pi B)A + 1/2\left[J_0(2\pi B) + J_2(2\pi B)\right]A_1;$$
$$a_2 = J_2(2\pi B)A + 1/2\left[J_1(2\pi B) + J_3(2\pi B)\right]A_1;$$
.....
$$b_1 = -J_1(2\pi B)A + 1/2\left[J_0(2\pi B) + J_2(2\pi B)\right]A_1$$
$$b_2 = J_2(2\pi B)A - 1/2\left[J_1(2\pi B) + J_3(2\pi B)\right]A_1$$
.....

and $J_i(2\pi B)$ denote Bessel functions. It is clear that by means of modulation we get the whole system of frequences of the type:

$$\alpha \pm k\beta, \quad k = 0, 1, 2 \ldots$$

with different amplitudes.

We are of the opinion that many of the observed phenomena can be regarded as the effect of the high frequency oscillation α with a suitable modulation in low frequency β .

The above considerations can be applied to the following stars:

1. The long period variations of RR Lyr according to A.M. Fringant [1] can be regarded as amplitude and frequency modulation: $\alpha = \frac{1}{P_0}$, $\beta = \frac{1}{P_1}$, $P_1 = 72\, P_0$.

2. According to one of our publications [2] the variations of 12 (DD) Lac can be described not by 2 or more short periods but by means of one short period P_s with the modulation in a long period P_L. The modulation exhibits the character of amplitude and frequency modulation and the long period P_L was found in the variations of many physical properties of the star.

3. The variations of ν Eri according to A. van Hoof [3] are represented by means of a great number of short periods. Some of them can be again regarded as the result of the modulation of the fundamental frequency $\alpha = \frac{1}{P_0} = \omega_0 = \omega_{24}$:

			β
$\alpha - 2\beta$	$= \omega_{02}$	= 5,6375	
$\alpha - \beta$	$= \omega_{13}$	= 5,7019	0,0644
α	$= \omega_0$	= 5,7634	0,0615
$\alpha + \beta$	$= \omega_{35}$	= 5,8275	0,0641

The mean value of $\beta = 0,0633$ corresponds to the long period $P_L = 15^d\!.790$ and the value of 2β to the period $P_b = 7^d\!.895$. The last one is equal to the beat period given in paper [3]. Probably also other frequences can be added to the same scheme:

$$\alpha + 2\beta = \omega_{46} = 5,8994$$
$$\alpha + 3\beta = \omega_f' = 5,9439$$
$$\alpha - 3\beta = \omega_f = 5,5866.$$

We can remark that this method of representation of complicated variations observed in some variable stars (e. g. with the Blashko-effect) requires from the theoretical point of view the explanation of only two frequences α and β.

References:

[1] Anne-Marie Fringant - Journal des Observat. 44. p. 187, 1961.

[2] A. Opolski and T. Ciurla - Acta Astronomica 11 p. 231, 1961.

[3] A. van Hoof - Z. Astrophys 55 p. 106, 1961.

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